# Notes for Calculus Lifesaver

**CHAPTER 1**

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1. Functions

* polynomials 多项式; exponentials 指数; logarithms 对数; trigonometric 三角法的, 三角学的
* [**R**](https://en.wikipedia.org/wiki/Real_number): the set of all real numbers: The real numbers include all the rational numbers **[Q]**, such as the integer **[Z]**[N] −5 and the fraction 4/3, and all the irrational numbers, such as √2 (1.41421356..., the square root of 2, an irrational algebraic number). Included within the irrationals are the transcendental numbers, such as π (3.14159265...). See **Figure 1**.

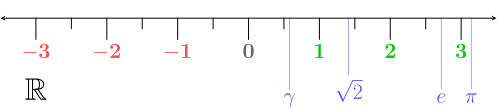


Figure 1. **[R]**: Real number; **[Q]**: rational numbers; **[Z]**: integers; **[N]**: natural numbers

1. Interval notation

* {*x*: 2≤ *x* < 5} = [2, 5)

1. Finding the domain (like *x*)

* The denominator of a fraction can't be zero. =! 0
* You can't take the square root (or fourth root, sixth root, and so on) of a negative number. ≥ 0
* You can't take the logarithm of a negative number or of 0. > 0
* (-8, 13] \ {2}: the domain is the set (-8; 13] except for the number 2.

1. Finding the range using the graph

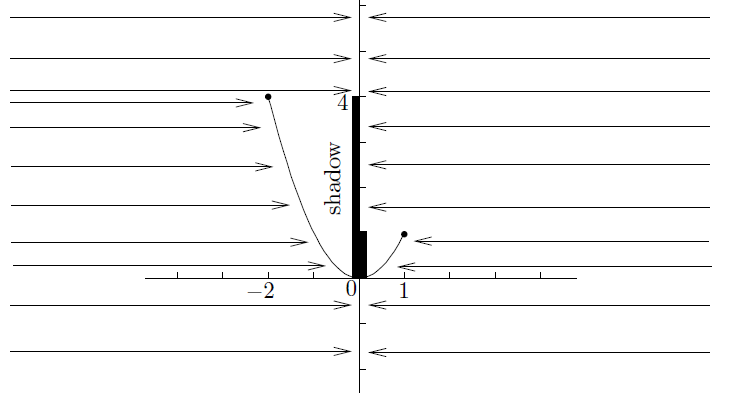


Figure 2. the range is [0, 4]

* Remember, the codomain of any function we look at will always be the set of all real numbers.
* codomain ≠ range.

1. The vertical line test

* How to judge whether it is a function? -> check the vertical lines: whether two or more points on the graph can lie on the same vertical line, see **Figure 3 and 4**.

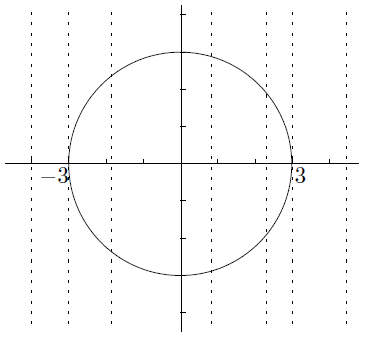


Figure 3. Not a function ()

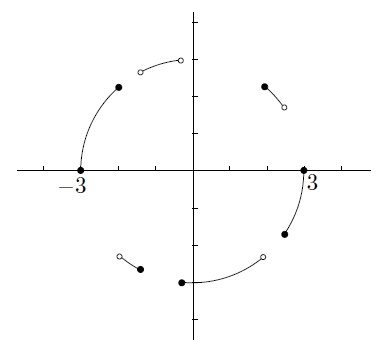


Figure 4. A function

1. Inverse Functions

* Start with a function *f* such that for any *y* in the range of *f*, there is exactly one number *x* such that *f*(*x*) = *y*. That is, different inputs give different outputs. Now we will define the inverse function *f*-1.
* The domain of *f*-1 is the same as the range of *f*.
* The range of *f*-1 is the same as the domain of *f*.
* The value of *f*-1(*y*) is the number *x* such that *f*(*x*) = *y*. So,

if *f*(*x*) = *y*; then *f*-1(*y*) = *x*.

1. The horizontal line test

* How to judge whether the function has an inverse function? -> check the horizontal lines: whether even one horizontal line intersects the graph more than once, see **Figure 5 and 6**.

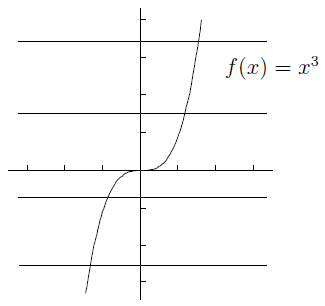


Figure 5. Inverse function existed

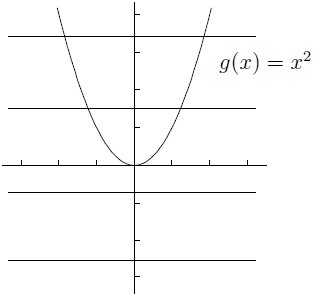


Figure 6. Inverse function doesn't exist

1. Finding the inverse

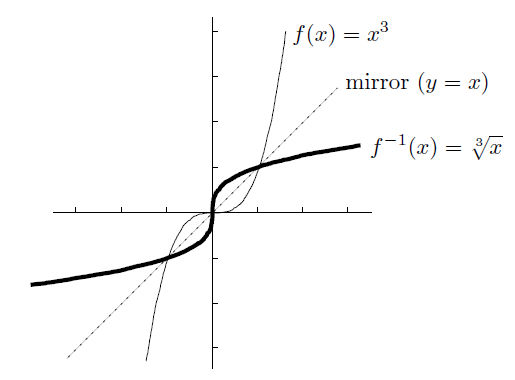


Figure 7. Draw the line y=x to find the inverse

1. Inverses of inverse functions

If the domain of a function *f* can be restricted so that *f* has an inverse *f*-1, then

* *f*(*f*-1(*y*)) = *y* for all *y* in the range of *f*; but
* *f*-1(*f*(*x*)) may not equal *x*; in fact, *f*-1(*f*(*x*)) = *x* only when *x* is in the restricted domain.

1. Composition of Functions

* *f*(x) = *h*(*g*(*x*)) can be expressed as *f* = *h* O *g*, so

*f*(x) = *m*(*k*(*j*(*h*(*g*(*x*))))) can write *f* = *m* O *k* O *j* O *h* O *g*

1. Odd and Even Functions

* Even Functions: *f*(-*x*) = *f*(*x*)
* Odd Functions: *f*(-*x*) = -*f*(*x*)
* Remember, odd functions must pass through the origin if they are defined at 0
* The product of two odd functions is always an even function, the product of two even functions is always even, and also that the product of an odd and an even function must be odd

1. Graphs of Linear Functions

* *f*(*x*)=*mx*+*b*, the slope is *m*, and the y-intercept is *b*. See **Figure 8**.

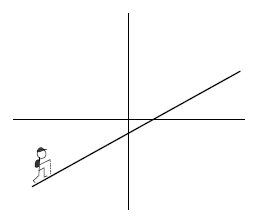


Figure 8. f(x)=mx+b

* the *point-slope* form of a linear function

If a line goes through (*x*0, *y*0) and has slope *m*,

then its equation is .

1. Common Functions and Graphs

* **Polynomials:** these are functions built out of nonnegative integer powers of *x*. You start with the building blocks 1, *x*, *x2*, *x*3, and so on, and you are allowed to multiply these basic functions by numbers and add a finite number of them together.
* The highest number *n* such that *x*n has a nonzero coefficient is called the *degree* of the polynomial.

where *a*n is the coefficient of *xn*, *a*n-1 is the coefficient of *x*n-1, and so on down to *a*0, which is the coefficient of 1.

* **Quadratics:** , it has two, one, or zero (real) roots, depending on the sign of the *discriminant* . , There are three possibilities. If , then there are two roots; if , there is one root (called a *double root*); and if , then there are no roots. In the first two cases, the roots are given by
* An important technique for dealing with quadratics is *completing the square*, like
* **Rational functions:** these are functions of the form

where *p* and *q* are polynomials. Some simplest examples are **Figure 9**:

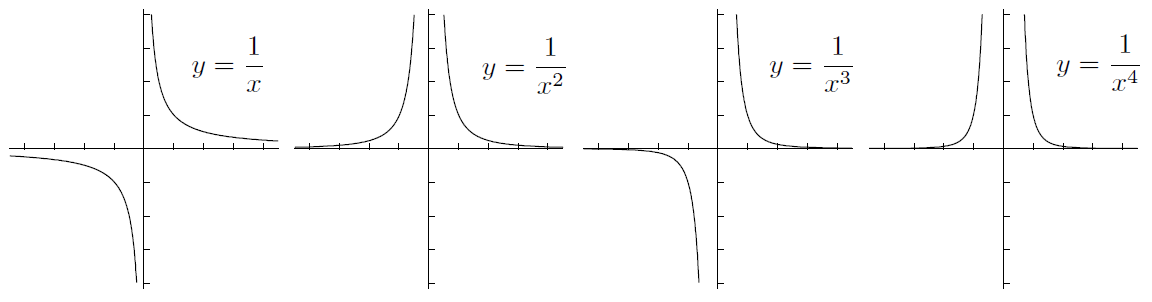


Figure 9. Some simplest rational functions

* **Exponentials and logarithms:** exponentials are , the domain is the whole real line, the *y-intercept* is 1, the range is , and there is a horizontal asymptote on the left or right at *y*=0. Since it satisfies the horizontal line test, there is an inverse function: the base *b* logarithm, which is written . The range is all of , and there’s a vertical asymptote at *x*=0. See **Figure 10**:

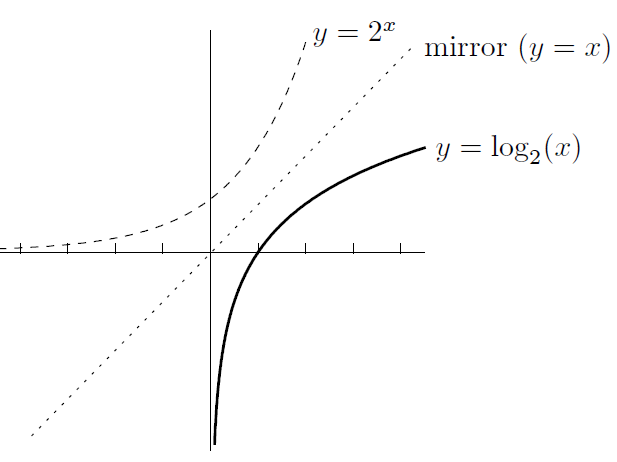


Figure 10. Base b is 2

**CHAPTER 2**

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1. The Basics

* circumference: 圆周, 周长, 胸围; right-angled: 直角的; hypotenuse: 直角三角形的斜边; reciprocal: 互惠的, 相互的, 倒数的, 彼此相反的;
* The circumference of a circle of radius 1 unit is 2π units, see **Figure 11**:

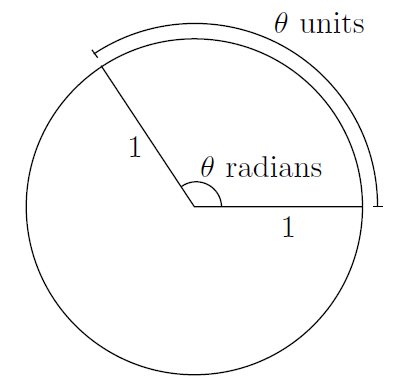


Figure 11. A circle of radius 1 unit

* The transfer formula between radians and degrees:
* A right-angled triangle **Figure 12**:

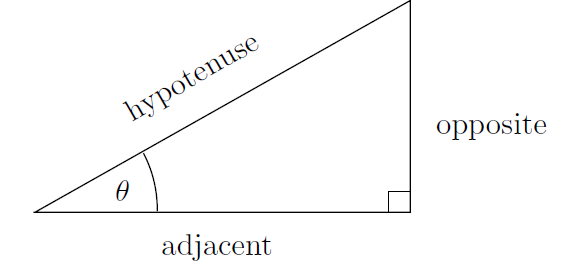


Figure 12. A right-angled triangle

We’ll also be using the reciprocal functions csc, sec, and cot:

* A nice table **Figure 13**:

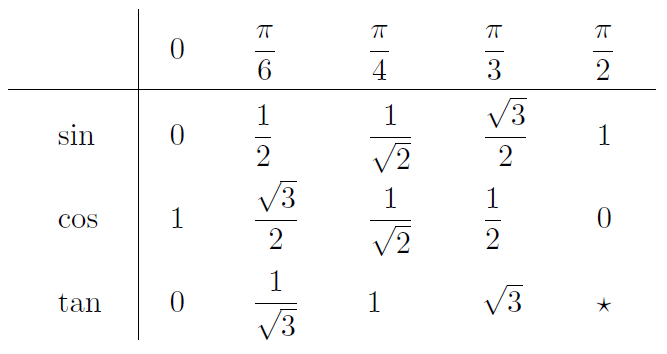


Figure 13. A nice table (the star means that tan(π/2) is undefined)

1. Extending the Domain of Trig Functions
2. The ASTC method

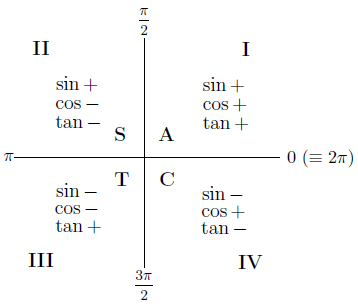


Figure 14. The ASTC method (All Sin Tan Cos)

1. The Graphs of Trig Functions

* (period 2π, odd), see **Figure 15:**

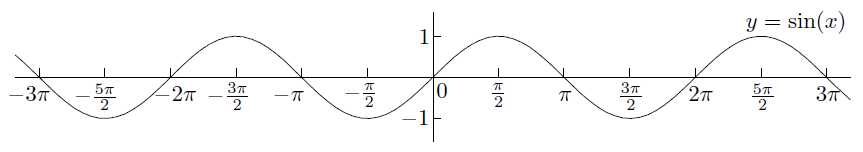


Figure 15. y=sin(x)

* (period 2π, even), see **Figure 16:**

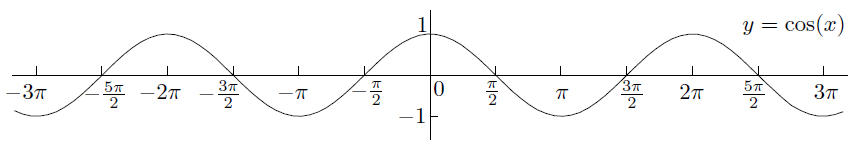


Figure 16. y=cos(x)

* (period π, odd), see **Figure 17:**

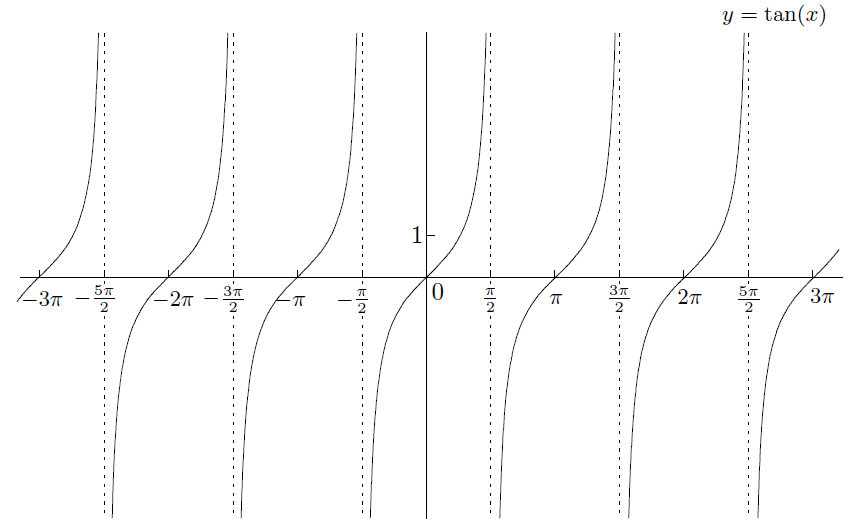


Figure 17. tan(x)

* (period 2π, even), see **Figure 18:**

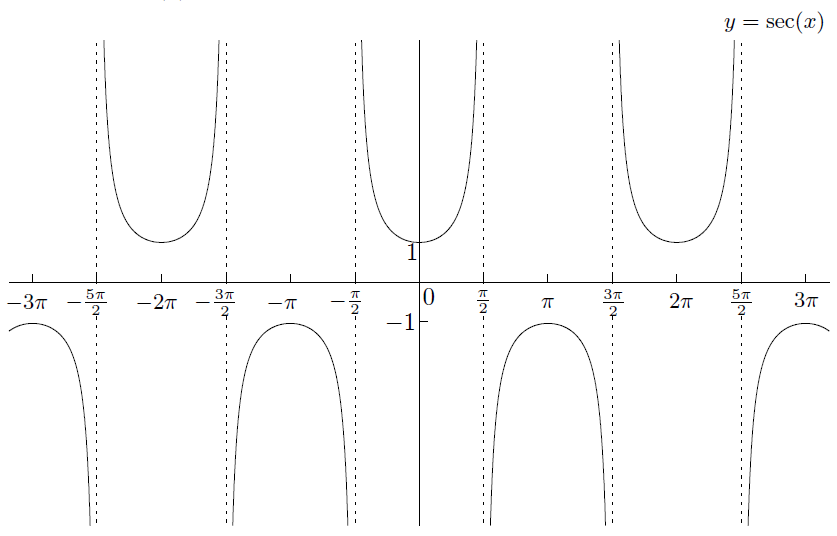


Figure 18. sec(x)

* (period 2π, odd), see **Figure 19:**

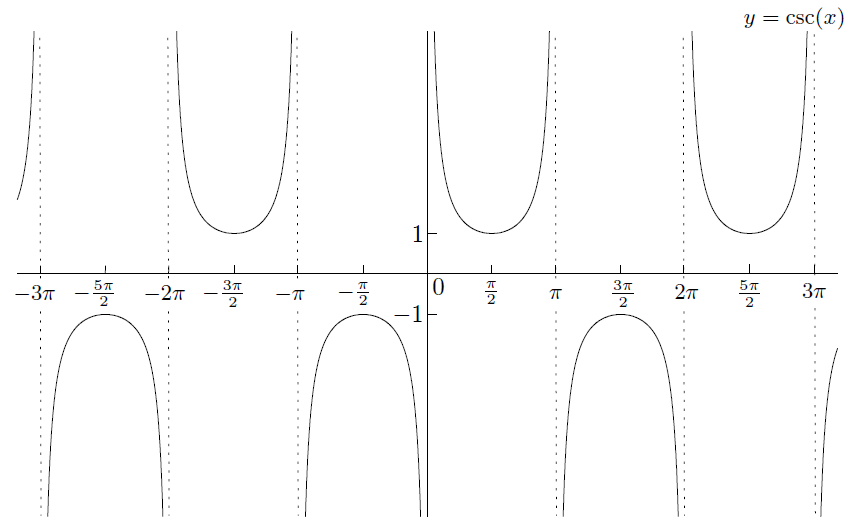


Figure 19. csc(x)

* (period π, odd), see **Figure 20:**

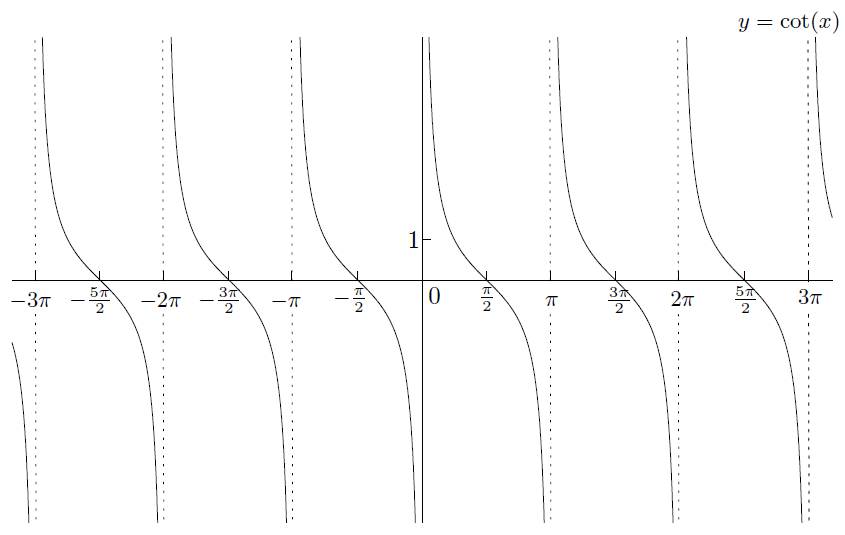


Figure 20. cot(x)

1. Trig Identities

* **:**

or

Specifically, you should remember that

The double-angle formulas are